Exercise 26

Find the derivative of the function.

$$s(t) = \sqrt{\frac{1+\sin t}{1+\cos t}}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
s'(t) = \frac{ds}{dt} &= \frac{d}{dt}\left[\left(\frac{1+\sin t}{1+\cos t}\right)^{1/2}\right] \\
&= \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-1/2} \cdot \frac{d}{dt}\left(\frac{1+\sin t}{1+\cos t}\right) \\
&= \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-1/2} \cdot \frac{\left[\frac{d}{dt}(1+\sin t)\right](1+\cos t) - \left[\frac{d}{dt}(1+\cos t)\right](1+\sin t)}{(1+\cos t)^2} \\
&= \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-1/2} \cdot \frac{(\cos t)(1+\cos t) - (-\sin t)(1+\sin t)}{(1+\cos t)^2} \\
&= \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-1/2} \cdot \frac{\cos t + \sin t + \cos^2 t + \sin^2 t}{(1+\cos t)^2} \\
&= \frac{1}{2}\left(\frac{1+\sin t}{1+\cos t}\right)^{-1/2} \cdot \frac{\cos t + \sin t + 1}{(1+\cos t)^2} \\
&= \frac{\cos t + \sin t + 1}{2\sqrt{(1+\sin t)(1+\cos t)^3}}
\end{aligned}$$